# Learning Hierarchical Probabilistic Logic Programs

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## Probabilistic Logic Programming (PLP) and Hierarchical PLP

- Probabilistic logic programming is a powerful tool for reasoning with **uncertain** relational models.
- Learning probabilistic logic programs is expensive due to the high cost of inference.
- We consider a restriction of the language of Logic Programs with Annotated Disjunctions called hierarchical PLP in which clauses and predicates are hierarchically organized.
- Generic clause $C$:

$$C = p(\vec{X}) : \pi :- \phi(\vec{X}, \vec{Y}), b_1(\vec{X}, \vec{Y}), \ldots, b_m(\vec{X}, \vec{Y})$$

where $\phi(\vec{X}, \vec{Y})$ is a conjunction of literals for the input predicates.

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## Generic Hierarchical PLP: https://rdcu.be/cmCIA

$$
\begin{aligned}
C_1 = r(\vec{X}) : \pi_1 &:- \phi_1, b_{1\_1}, \ldots, b_{1\_m_1} \\
&\ldots \\
C_n = r(\vec{X}) : \pi_n &:- \phi_n, b_{n\_1}, \ldots, b_{n\_m_n} \\
C_{1\_1\_1} = r_{1\_1}(\vec{X}) : \pi_{1\_1\_1} &:- \phi_{1\_1\_1}, b_{1\_1\_1\_1}, \ldots, b_{1\_1\_1\_m_{111}} \\
&\ldots \\
C_{1\_1\_n_{11}} = r_{1\_1}(\vec{X}) : \pi_{1\_1\_n_{11}} &:- \phi_{1\_1\_n_{11}}, b_{1\_1\_n_{11}\_1}, \ldots, b_{1\_1\_n_{11}\_m_{11n_{11}}} \\
&\ldots \\
C_{n\_1\_1} = r_{n\_1}(\vec{X}) : \pi_{n\_1\_1} &:- \phi_{n\_1\_1}, b_{n\_1\_1\_1}, \ldots, b_{n\_1\_m_{n11}} \\
&\ldots \\
C_{n\_1\_n_{n1}} = r_{n\_1}(\vec{X}) : \pi_{n\_1\_n_{n1}} &:- \phi_{n\_1\_n_{n1}}, b_{n\_1\_n_{n1}\_1}, \ldots, b_{n\_1\_n_{n1}\_m_{n1n_{n1}}} \\
&\ldots
\end{aligned}
$$

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## Example

- UW-CSE domain, the objective is to predict the "advised_by" relation

$C_1$ = *advised_by*$(A, B) : 0.3 :-$
*student*$(A)$, *professor*$(B)$, *project*$(C, A)$, *project*$(C, B)$, $r_{11}(A, B, C)$.

$C_2$ = *advised_by*$(A, B) : 0.6 :-$
*student*$(A)$, *professor*$(B)$, *ta*$(C, A)$, *taught_by*$(C, B)$.

$C_{111} = r_{11}(A, B, C) : 0.2 :-$
*publication*$(D, A, C)$, *publication*$(D, B, C)$.

- $r$ = *advised_by*(*harry*, *ben*) where *harry* is a student, *ben* is a professor, they have two joint courses, two joint projects and two joint publications from each jointed project.

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## Parameter learning for HIerarchical probabilistic Logic programs

- Given a HPLP $T$ with parameters $\Pi$, and a training set $E = \{e_1, \ldots, e_M, \text{not } e_{M+1}, \ldots, \text{not } e_N\}$ **PHIL** finds the values of $\Pi$ that maximize the log likelihood:

$$\arg\max_{\Pi} \sum_{i=1}^{M} \log P(e_i) + \sum_{i=M+1}^{N} \log(1 - P(e_i)) \quad (1)$$

$P(e_i)$ is the probability of example $e_i$.

- Or equivalently minimizing the sum of *cross entropy errors* for all the examples $err_i = -y_i \log(p_i) - (1 - y_i) \log(1 - p_i)$
- Deep PHIL (DPHIL) and Expectation Maximization PHIL (EMPHIL) learn the parameters using Gradient Descent and Expectation Maximization.

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## Structure LEArning of Hierarchical Probabilistic logic programming

Given a set of interpretations, each containing positive and negative examples for a target predicate and facts for input predicates,
$I = \{e_1, \ldots, e_M, \text{not } e_{M+1}, \ldots, \text{not } e_N\}$, **SLEAHP** finds an HPLP with parameters $\Pi$ that maximizes the (log) likelihood

$$LL = \arg\max_{\Pi} \sum_{k}^{Nl} (\sum_{i=1}^{M} \log P(e_{i_k}) + \sum_{i=M+1}^{N} \log(1 - P(e_{i_k}))) \quad (2)$$

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## Structure Learning: SLEAHP algorithm

1. **Input**: a set of interpretations and a target predicate (examples).
2. Generate a set of bottom clauses from a language bias.
3. Generate a program tree from the bottom clauses.
4. **Randomly** generate an initially **large** HPLP from the tree.
5. Apply a regularized version of PHIL on the HPLP.
6. Discard Clauses with very small probability values.

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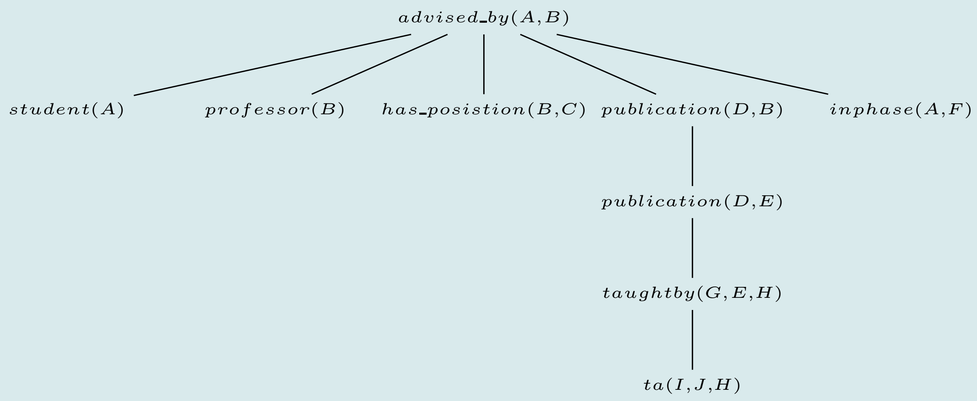
## Structure Learning: program tree from a bottom clause

*advised_by*$(A, B) :-$ *student*$(A)$, *professor*$(B)$, *has_position*$(B, C)$, *publication*$(D, B)$, *publication*$(D, E)$, *inphase*$(A, F)$, *taughtby*$(G, E, H)$, *ta*$(I, J, H)$.

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## Structure Learning: initial HPLP generated from the tree

*advised_by*$(A, B) : 0.5 :-$ *student*$(A)$.
*advised_by*$(A, B) : 0.5 :-$ *professor*$(B)$.
*advised_by*$(A, B) : 0.5 :-$ *has_position*$(B, C)$.
*advised_by*$(A, B) : 0.5 :-$ *publication*$(D, B)$, *hidden_1*$(A, B, D)$.
*advised_by*$(A, B) : 0.5 :-$ *inphase*$(A, E)$.
*hidden_1*$(A, B, D) : 0.5 :-$ *publication*$(D, F)$, *hidden_11*$(A, B, D, F)$.
*hidden_11*$(A, B, D, F) : 0.5 :-$ *taughtby*$(G, F, H)$, *hidden_111*$(A, B, D, F, G, H)$.
*hidden_111*$(A, B, D, F, G, H) : 0.5 :-$ *ta*$(I, J, H)$.

- Learning HPLPs on web browser: **phil on SWISH**
- Link: https://cplint.eu/e/phil/phil_examples.swinb
- Manual: https://arnaudfadja.github.io/phil/
- Paper: https://rdcu.be/cmCIA

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## Experiments: Average time for PHIL

| Time | Mutagenesis | Carcinogenesis | Mondial | UWCSE | PAKDD15 |
|---|---|---|---|---|---|
| DPHIL | *2.8573* | 178.2680 | 265.4160 | **0.0884** | 34.8170 |
| $DPHIL_1$ | 5.2059 | 177.4270 | 311.3280 | 0.2910 | 20.5704 |
| $DPHIL_2$ | 5.4450 | 88.5100 | 301.1392 | 0.2214 | 4.9517 |
| EMPHIL | 4.4430 | *106.5540* | 270.6688 | 0.2890 | 6.6106 |
| $EMPHIL_1$ | 4.8940 | *181.0890* | 317.4202 | 1.0000 | 7.0691 |
| $EMPHIL_2$ | 5.0046 | 146.8440 | **245.3830** | *0.8372* | 6.6334 |
| $EMPHIL_{B_1}$ | 2.6478 | 85.3210 | 248.1978 | 0.1572 | 6.0990 |
| $EMPHIL_{B_2}$ | **1.5820** | **80.4270** | *261.3612* | 0.1266 | 8.8722 |
| $EMPHIL_{B_3}$ | 1.4937 | 94.6850 | 274.9598 | 0.1190 | 6.0985 |
| EMBLEM | 125.62 | - | - | 0.9666 | 154.51 |
| ProbLog2 | 722.00 | 38685.00 | 1607.60 | 161.40 | 596.90 |
| Tuffy | - | - | - | - | - |

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## Experiments: Average time for SLEAHP

| Time | Mutagenesis | Carcinogenesis | Mondial | UWCSE | PAKDD15 |
|---|---|---|---|---|---|
| $SLEAHP_{G_1}$ | 41.8250 | **48.7600** | *59.5054* | *219.6410* | 192.4396 |
| $SLEAHP_{G_2}$ | 47.1344 | 10524.0900 | **14.0470** | *194.9706* | *162.9938* |
| $SLEAHP_{E_1}$ | 48.0152 | *303.0570* | 60.8316 | 387.6650 | 151.7217 |
| $SLEAHP_{E_2}$ | *45.9245* | 92.3820 | 61.0996 | 312.2604 | 68.2432 |
| $SLEAHP_B$ | **13.1478** | 1399.0090 | 14.6698 | 295.6734 | **61.5347** |
| SLIPCOVER | 74610.7000 | 17419.4500 | 650.3630 | **141.3600** | 242.7077 |
| PROBFOIL+ | 1726.6000 | 15433.0000 | - | - | - |
| MLN-BC | 91.7700 | 59.73500 | 56.5007 | 376.2356 | * |
| MLN-BT | 360.8563 | 87.5020 | 53.5568 | 891.4226 | 915.5794 |
| RDN-B | 183.7140 | 61.5000 | 192.0487 | 501.1176 | 793.7661 |

- PHIL and SLEAHP Provide similar performance w.r.t SOTA in terms of area under the ROC and PR curves but appear to be scalable.

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