A Semantics for Hybrid Probabilistic Logic Programs with Function Symbols

Damiano Azzolini, Fabrizio Riguzzi, and Evelina Lamma
University of Ferrara
https://ml.unife.it/

University of Ferrara

Keywords: **Hybrid Probabilistic Logic Programs, Semantics**

Probabilistic Logic Programming

- **Probabilistic Logic Programming** (PLP) is a useful paradigm for encoding models characterized by complex relations heavily depending on probability. Simple example:

```
0.6::no_sleep.
0.7::too_much_work.
tired:- no_sleep.
tired:- too_much_work.
```

- Find the probability of queries: *inference*
- When there are both discrete and continuous random variables, or there are function symbols, the task is complicated, and the semantics may not be straightforward

Probabilistic Logic Programs with Function Symbols I

Game with three cards: ace of spades, ace of clubs, and ace of hearts. The player looses when he/she picks the ace of hearts.

```
1/3 :: spades(X).
1/2 :: clubs(X).
pick(0,spades) :- spades(0).
pick(0,clubs) :- \+ spades(0), clubs(0).
pick(0,hearts) :- \+ spades(0), \+ clubs(0).
pick(s(X),spades):- \+ pick(X,hearts), spades(s(X)).
pick(s(X),clubs):- \+ pick(X,hearts), \+ spades(s(X)),
                   clubs(s(X)).
pick(s(X),hearts):- \+ pick(X,hearts), \+ spades(s(X)),
                    \+ clubs(s(X)).

at_least_once_spades :- pick(_,spades).
```

- Is the probability of `at_least_once_spades` Well-defined?
- The query `at_least_once_spades` has an infinite number of groundings, how to compute its probability?

Probabilistic Logic Programs with Function Symbols II

The probability is indeed Well-defined, and can be computed by considering *pairwise incompatible covering set of explanations*, in this way:

$$\begin{aligned} P(\text{at\_least\_once\_spades}) &= \frac{1}{3} + \frac{1}{3} \cdot \left(\frac{2}{3} \cdot \frac{1}{2}\right) + \frac{1}{3} \cdot \left(\frac{2}{3} \cdot \frac{1}{2}\right)^2 + \dots \\ &= \frac{1}{3} + \frac{1}{3} \cdot \left(\frac{1}{3}\right) + \frac{1}{3} \cdot \left(\frac{1}{3}\right)^2 + \dots \\ &= \frac{1}{3} \cdot \frac{1}{1 - \frac{1}{3}} = \frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2} \end{aligned}$$

What happens if we have both continuous random variables and constraints?

Hybrid Probabilistic Logic Programs I - Example

Gaussian mixture encoded in `cplint` hybrid programs:

```
0.6::h.
heads :- h.
tails :- \+ h.
g(X) : gaussian(X, 0, 1).
h(X) : gaussian(X, 5, 2).
mix(X) :- heads, g(X).
mix(X) :- tails, h(X).
mix :- mix(X), X > 2.
```


Hybrid Probabilistic Logic Programs II

- Given the previous program, the argument `X` of `mix(X)` follows a distribution that is a mixture of two Gaussians, one with mean 0 and variance 1 with probability 0.6, and one with mean 5 and variance 2 with probability 1 - 0.6 = 0.4. We can ask for the probability of `mix`
- How about discrete and continuous random variables and function symbols?
- What happens if we add a continuous random variable and a constraint to the body of the clauses `pick/2` of the previous game example?

Hybrid Probabilistic Logic Programs III

Variation of the game example, with another requirement: the player spins a wheel, and the game continues only if the axis is in the range $]\pi, 2\pi]$.

```
...
angle(_,X) : uniform_dens(X,0,6.28).
...
pick(0,spades) :- spades(0), angle(0,V), V > 3.14.
...
pick(s(X),spades):- \+ pick(X,hearts), spades(s(X)),
    angle(s(X),V), V > 3.14.
...
```

- Countable set X of continuous random variables (identified by 0, $s(0)$, ...): each element has a range $[0, 6.28]$
- Countable set Y of discrete random variables
- The probability is still Well-defined, and can be computed by considering *mutually disjoint covering set of worlds* $\omega = \omega_1 \cup \omega_2 \cup \dots$

$$\begin{aligned} \omega_0 = \{(w_\mathrm{X}, w_\mathrm{Y}) \mid\ & w_\mathrm{X} = (x_0, x_1, \dots), w_\mathrm{Y} = (y_0^c, y_0^s, y_1^c, y_1^s, \dots), \\ & x_0 \in]\pi, 2\pi], y_0^s = 1\} \\ \omega_1 = \{(w_\mathrm{X}, w_\mathrm{Y}) \mid\ & w_\mathrm{X} = (x_0, x_0, \dots), w_\mathrm{Y} = (y_0^c, y_0^s, y_1^c, y_1^s, \dots), \\ & x_0 \in]\pi, 2\pi], y_0^s = 0, y_0^c = 1, x_1 \in]\pi, 2\pi], y_1^s = 1\} \\ & \dots \end{aligned}$$

We get $\frac{1}{3} \cdot \frac{1}{2} \cdot \sum_{i=0}^{\infty} (\frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{2})^i = \frac{1}{6} \cdot \sum_{i=0}^{\infty} (\frac{1}{6})^i = \frac{1}{6} \cdot \frac{6}{5} = \frac{1}{5}$ as probability for the query `at_least_once_spades`.

- Well-definedness proof idea: we define some operators and prove that they are sound and complete

Paper Reference

[1] Damiano Azzolini, Fabrizio Riguzzi, and Evelina Lamma. A semantics for hybrid probabilistic logic programs with function symbols. Artificial Intelligence, 294:103452

Department of Engineering Ferrara
Damiano Azzolini
damiano.azzolini@unife.it
Department of Engineering
University of Ferrara

https://ml.unife.it/