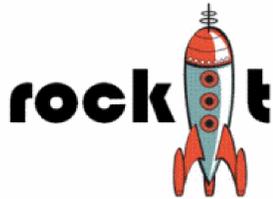


# Backpropagating through Markov Logic Networks

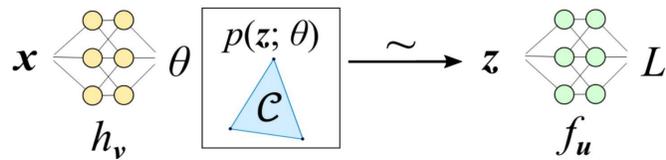
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## Overview

Markov Logic Networks [1]



Implicit Maximum Likelihood Estimation (IMLE) [2]



High-dimensional Noisy Data

**Kristin** and her **son Justin** went to visit her **mother Carol** on a nice Sunday afternoon. They went out for a movie together and had a good time.

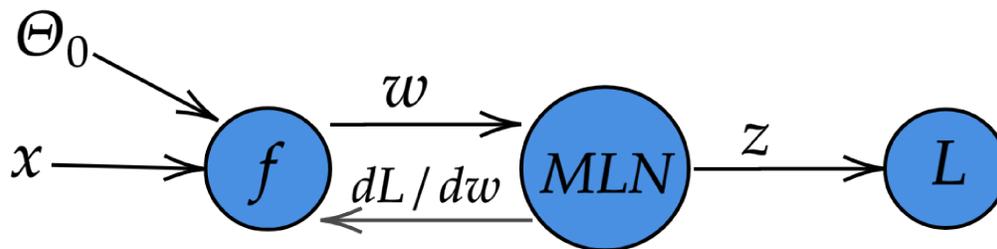
Q: How is **Carol** related to **Justin** ?

A: Carol is the **grandmother** of Justin

(from CLUTRR [3])

## General

By using what is already there we can build a simple end-to-end trainable NeSy model.



## Model

MLN: 
$$p(\mathbf{z}; \mathbf{w}) = \frac{1}{Z} \exp \left\{ \sum_i w_i z_i \right\}$$

Adding a neural module:

$$\mathbf{w} = f(\mathbf{x}; \Theta_0) \quad z \sim p(\mathbf{z}; \mathbf{w})$$

## Gradients

Perceptron Rule: 
$$\frac{dL}{d\mathbf{w}} = \text{MAP}(\mathbf{w}) - \mathbf{z}$$

IMLE [2]: 
$$\frac{dL}{d\mathbf{w}} = \frac{1}{\lambda} \left[ \text{MAP}(\mathbf{w} + \epsilon) - \text{MAP} \left( \mathbf{w} - \lambda \frac{dL}{dz} + \epsilon \right) \right]$$

\*Using no Gumbel noise leads to [4]

## Data

- Text datasets governed by rules
- Schema Dataset based on Description Logics
- CLUTRR dataset [3]
- MLN MAP state used as final predictions

## Result/Conclusion

- Outperform neural baselines
- Noise perturbations are necessary for training
- Atoms/Structure needs to be predefined

## References

- [1] M. Richardson, P. Domingos, Markov logic networks, in: Machine learning, volume 62, Springer, 2006, pp. 107–136
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- [4] M. V. Pogančić, A. Paulus, V. Musil, G. Martius, M. Rolinek, Differentiation of blackbox combinatorial solvers, in: International Conference on Learning Representations, 2020.